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Pressure Loss Modulus Correlation for Δp Across Uniformly Distributed-Loss Devices

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A dimensionless group, called a pressure loss modulus (N_{PL}), is introduced that, in conjunction with an appropriately defined Reynolds number, is of considerable engineering utility in correlating steady-state Δp vs flow calibration data and subsequently as a predictor, using the same or a different fluid, in uniformly distributed pressure loss devices. It is particularly useful under operation in the transition regime. Applications of this simple bivariate correlation to three diverse devices of particular interest for small liquid rocket engine fluid systems are discussed: large *L/D* capillary tube restrictors, packed granular catalyst beds, and stacked vortex-loss disk restrictors.

Nomenclature

A = cross-sectional area
D = diameter
d = differential operator
f = friction factor (dimensionless) in Fanning, or Darcy, equation
G = mass flux per unit cross-sectional area
g_c = universal force-mass gravitational conversion constant (not needed with SI units)
K, k = proportionality constants (dimensionless)
L = length
$\bar{M}$ = average molecular weight of gas
m = exponent (dimensionless)
N_{PL} = pressure loss modulus (dimensionless)
N_{Re} = Reynolds number (dimensionless)
n = exponent (dimensionless)
p = average static pressure
R = universal gas constant
r = exponent (dimensionless)
S = surface area
S_S = specific surface of packing (surface per unit volume)
$\bar{T}$ = average absolute temperature of gas
x = characteristic linear transverse dimension of flow passage
$\bar{z}$ = average compressibility factor
β = correction term to Fanning equation for a capillary
Δp = pressure drop
ε = void fraction of packing (dimensionless)
μ = viscosity
ν = number of stages in stack (dimensionless)
ρ = density
σ = average sphericity of particle (dimensionless)
$\phi(\,)$ = unspecified function
$\psi(\,)$ = unspecified function

Subscripts

e = effective
f = exposed to flow
g = granule
o = referred to total cross section (without packing)
p = passage minimum
S = one stage of a stack of disks
t = tube

Introduction

IN utilizing distributed-loss devices (such as tubing, metal and ceramic foams, stacked etched platelets, and granular packings) in fluid systems, one is frequently faced with the necessity of calibrating with one fluid under some convenient set of conditions, although the intended use involves a different fluid and a wide range of operating conditions. The question arises of how best to correlate the data to facilitate the conversion among fluids and to different operating conditions. The problem is further compounded when operation in the laminar/turbulent transition region is required, where the functional relationships are not directly amenable to analytic representation, and hence the form of the requisite density and viscosity corrections is uncertain.

In this article, a dimensionless pressure loss modulus parameter N_{PL} is introduced, which, together with the Reynolds number, forms a pair of engineering correlation parameters adequate to uniquely define the steady-state pressure-drop-vs-flow characteristic of any uniformly distributed-loss device over the laminar and turbulent flow regimes for (effectively) incompressible fluids. Examples of applications to specific devices, with appropriate interpretation of the constituent variables in each case, are subsequently discussed.

Approach

Given the conditions of a steady flow of an incompressible fluid through a distributed-loss device, wherein effects due to surface tension and body forces are negligible and density and viscosity are essentially constant, then for any geometrically similar flow passage, the classical fluid mechanics treatment (see, for instance, Massey[1] or Sabersky and Acosta[2]) by dimensional analysis yields the relationship

$$\frac{\rho x}{G^2}\frac{\mathrm{d}p}{\mathrm{d}L} = \phi\left[\frac{xG}{\mu}, \frac{L}{x}\right] \tag{1}$$

where the function ϕ must be determined experimentally. For the applications of interest in this analysis, assume that length does not enter the functional relationship as an independent variable; specifically assume that 1) the cross-sectional flow area is constant (or repetitive in the "length" dimension), 2) entrance effects are negligible, so that $\mathrm{d}p/\mathrm{d}L$ is constant in a flow passage and may be replaced by the corresponding ratio of finite increments, 3) flow passages are geometrically similar in cross section, and 4) the

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governing flow parameter is the maximum local mass flux G in the flow channel(s). Under these assumptions, and with the insertion of g_c to permit the use of engineering units, the foregoing relationship becomes more explicitly

$$\frac{g_c \rho x \Delta p}{G^2 L} = 2\phi\left[\frac{xG}{\mu}\right] \tag{2}$$

Equation (2) is, in fact, just a rearrangement of the well-established Fanning, or Darcy, equation for pipe flow if x is taken to be the pipe diameter D and ϕ is identified as the friction factor f. However, the objective in this instance is to isolate the flow parameter from the pressure drop parameter. Recognizing the group xG/μ as the Reynolds number N_{Re} and formally multiplying both members of Eq. (2) by its square yield

$$\frac{g_c \rho x^3 \Delta p}{\mu^2 L} = 2\left(\frac{xG}{\mu}\right)^2 \phi\left[\frac{xG}{\mu}\right] \tag{3}$$

Both sides of Eq. (3) are dimensionless. Note that nothing has as yet been assumed about the nature of the flow regime, and hence Eq. (3) is valid for laminar, transition, or turbulent flow. The nature of the function ϕ depends on the character of the flow, but the right-hand member is a unique function of Reynolds number only for a given device, that is,

$$\frac{g_c \rho x^3 \Delta p}{\mu^2 L} = 2N_{Re}^2 \phi(N_{Re}) = \psi(N_{Re}) \tag{4}$$

If the group on the left of Eq. (4) is defined as the dimensionless pressure loss modulus N_{PL},

$$N_{PL} \equiv \frac{g_c \rho x^3 \Delta p}{\mu^2 L}$$

Eq. (4) becomes simply

$$N_{PL} = \psi(N_{Re}) \tag{5}$$

Equation (5) indicates that providing the few mildly restrictive assumptions are met, experimental steady-state pressure drop data—taken with varying fluids under different conditions for a uniformly distributed-loss device—should all be groupable on a single curve using the parameters N_{PL} and N_{Re}. This curve can be used as a predictor throughout the flow regime spanned by the data. It obviates questions as to what exponents should be used in density and viscosity corrections, if a different fluid or temperature is of interest.

Frequently, the functional dependence of ϕ on Reynolds number can be adequately approximated by

$$\phi(N_{Re}) \approx K N_{Re}^{-m}$$

and Eq. (5) becomes

$$N_{PL} \approx K N_{Re}^{n} \tag{5a}$$

where $n = 2-m$. Form (5a) is particularly convenient because it plots linearly in bilogarithmic coordinates, thus facilitating limited extrapolations to predict behavior beyond the range studied experimentally. The applicability of form (5a) over the more general form (5) must, of course, be experimentally determined in any given instance.

Applications

In the sections that follow, the application of the foregoing generalized correlation technique to three specific distributed-loss devices, frequently used in small liquid rocket fluid systems, is discussed: a capillary tube operating in the transition regime, a granular-bed-type catalytic reactor, and a vortex-loss stacked-disk metering device.

Capillary Tube

A length of capillary tubing, with an inside diameter of 8.5 mil, was used as both an "orifice" and a flow metering device in a miniature rocket engine. The normal operating range was in the laminar/turbulent transition regime. The working fluid was hydrazine, but calibration was effected with water, and the N_{PL} vs N_{Re} correlation technique was used with excellent success. The characteristic x dimension used in both N_{Re} and N_{PL} is the internal tube diameter D_t, and L_t is the physical length of the tube. Thus,

$$N_{Re} \equiv \frac{D_t G}{\mu}$$

and

$$N_{PL} \equiv \frac{g_c \rho D_t^3 \Delta p}{\mu^2 L_t}$$

Riebling and Powell[3] have shown, through a detailed analysis of a capillary passage, that Eq. (2) should contain an L/D correction through an additional term β in the form

$$\frac{g_c \rho x \Delta p}{G^2 L} = 2(1+\beta)\phi(N_{Re}) \tag{6}$$

where β is a function of the passage L/D, entrance geometry, and N_{Re} for the flow regime in question. In this application, however, the tube L/D was large (~ 10^3), resulting in a range of β from 0.02 to 0.03 (« 1). Hence, the additional complexity of Eq. (6) is not warranted, and the correlation parameters derived from the simpler form of Eq. (2) were employed.

The experimental data, plotted in Fig. 1, fall on a single curve with minimal scatter, confirming the validity of the assumptions. The curve is nonlinear over most of the range of interest, as anticipated from the range of Reynolds numbers, indicating that Eq. (5a)

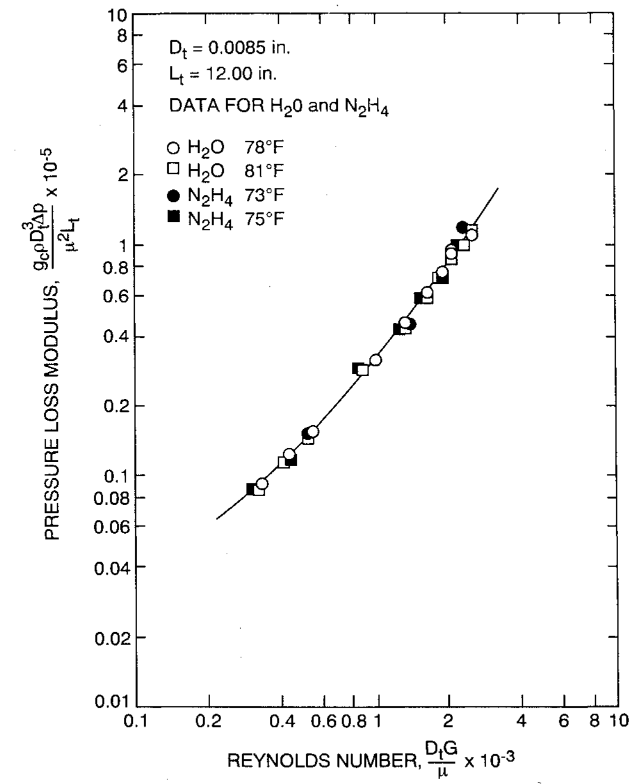

Fig. 1 Capillary tube correlation.

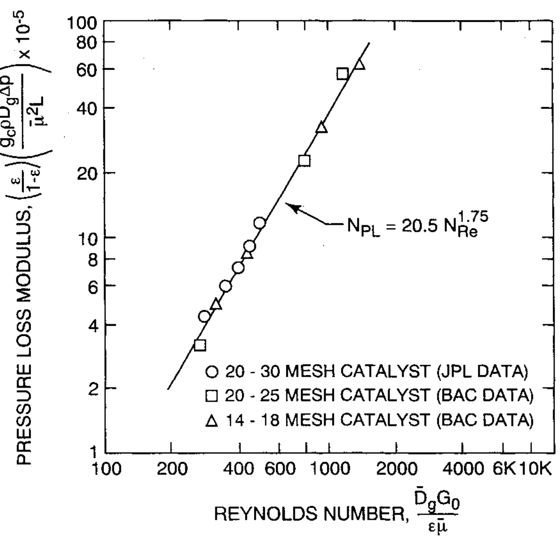

Fig. 2 Catalyst bed correlation.

does not apply. However, the curve itself is perfectly adequate for correlating the data and calculating desired flow rates, given Δp and the fluid properties.

Catalyst Bed

Catalyst beds packed with granular material have wide industrial application. One such is as a combustion chamber for monopropellant rocket engines and gas generators. The flow characteristics of the beds are of major importance. Data of this type have been correlated by other investigators[4] using semiempirical dimensional equations, and it was therefore of interest to compare the use of the correlating parameters described herein. For a packed bed, some interpretation is required in defining N_{Re} and N_{PL} appropriately to account for the packing characteristics. To aid in this interpretation, we temporarily write Eq. (2) in Fanning form. For pipes with $x = D$,

$$\Delta p = \phi(N_{Re})\frac{2G^2}{g_c\rho}\frac{L}{D}$$

and multiplying the numerator and denominator of the right member by $\pi D/4$ gives the alternative form

$$\Delta p = \frac{\phi(N_{Re})G^2}{2g_c\rho}\left[\frac{\pi D L}{(\pi/4)D^2}\right] = \frac{\phi(N_{Re})G^2}{2g_c\rho}\frac{S_f}{A_f} \tag{7}$$

Thus, the frictional pressure drop is proportional to the ratio of wetted surface area of the flow channel (pipe) to its transverse flow area. It is reasonable to assume that a similar proportionality holds for the packed bed. In terms of the bed void fraction ε, the transverse flow area is related to the total bed cross section by

$$A_f = \varepsilon A$$

and the actual mass flux is consequently related to the superficial mass flux, defined for the empty chamber, by

$$G = G_o/\varepsilon$$

The wetted surface of packing is given by the product of its specific surface and bulk volume

$$S_f = ALS_s$$

For quasispherical (sphericity, $\sigma \geq 0.7$) granules, the specific surface is related to the mean effective granule diameter $\bar{D}_g$ by

$$S_s = \frac{6(1-\varepsilon)}{\sigma\bar{D}_g}$$

The characteristic passage dimension x should be the mean hydraulic diameter of the interstices between granules. For monosize spherical particles, this dimension is directly proportional to the particle diameter, and for the quasispherical granules employed here this is still a reasonably good assumption. Hence assume

$$x = k\bar{D}_g$$

Substituting all of these relationships back into Eq. (7), rearranging, and multiplying through by $(\bar{D}_g/\mu)^2$ yields

$$\left(\frac{\varepsilon}{1-\varepsilon}\right)\left(\frac{g_c\rho\bar{D}_g^3\Delta p}{\mu^2 L}\right) = \frac{3}{\sigma}\left(\frac{\bar{D}_g G_o}{\varepsilon\mu}\right)^2\phi\left[\frac{k\bar{D}_g G_o}{\varepsilon\mu}\right] \tag{8}$$

For the packed bed, it is therefore logical on the basis of the groupings in Eq. (8) to define a modified Reynolds number and pressure loss modulus by

$$N_{PL} \equiv \left(\frac{\varepsilon}{1-\varepsilon}\right)\left[\frac{g_c\bar{\rho}\bar{D}_g^3\Delta p}{\bar{\mu}^2 L}\right]$$

and

$$N_{Re} \equiv \frac{\bar{D}_g G_o}{\varepsilon\bar{\mu}}$$

wherein the average gas properties

$$\bar{\rho} = \frac{\bar{p}\bar{M}}{\bar{z}R\bar{T}}, \qquad \bar{\mu} = \mu(\bar{T})$$

are used to allow for their variation through the bed.

Data from several reactors using catalysts of various mesh sizes are shown plotted in this form in Fig. 2, with Reynolds numbers ranging from about 200 to 2000. The scatter is greater than that of Fig. 1 and is probably due primarily to 1) difficulty in accurately measuring the small differential pressures across the beds in an environment of high absolute pressure and 2) small L/D for some of

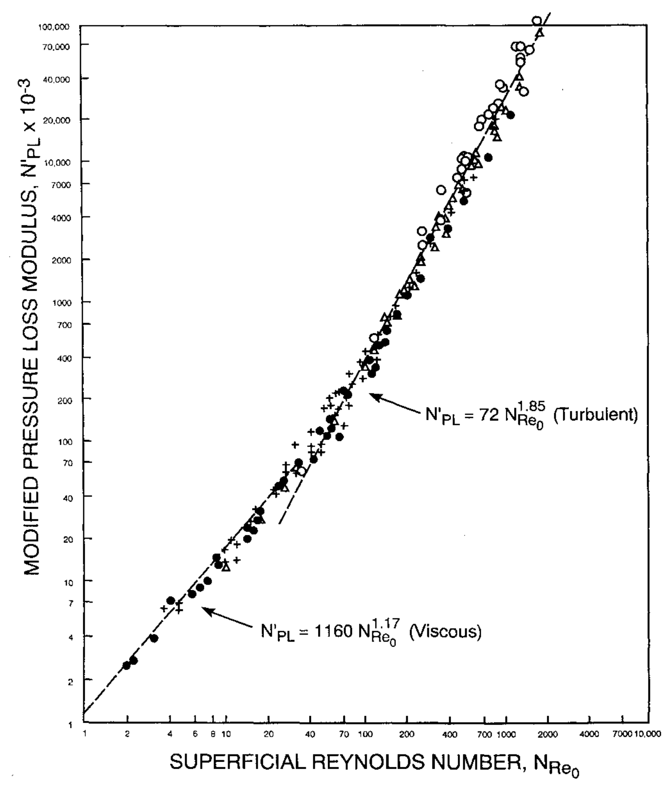

Fig. 3 Chilton and Coburn data for various packed towers plotted in N_{PL} vs N_{Re} format.

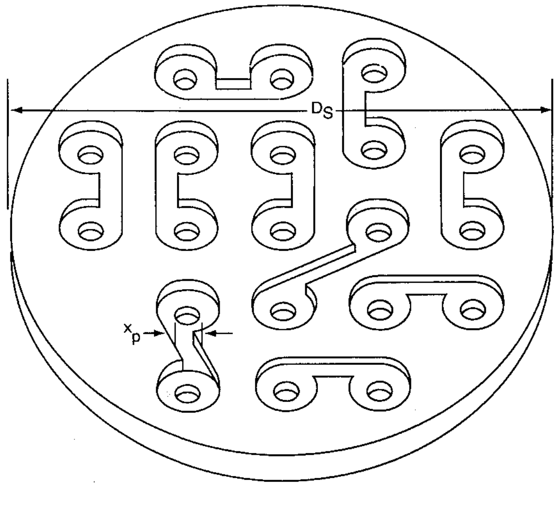

Fig. 4 "Lee Visco Jet" (one stage).

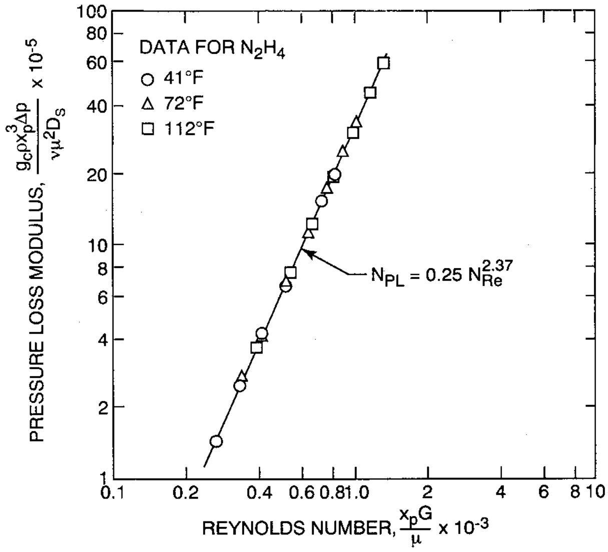

Fig. 5 Visco Jet correlation.

the beds, weakening the assumption of negligible entrance effects. Instrument error, compressibility and varying properties of the gas, and uncertainties in packing parameters also contribute to the scatter. The correlation is nevertheless good, and it appears that the data can be adequately represented by an equation of form (5a) with $K \approx 20.5$ and $n \approx 1.75$, which result may be compared directly with the pressure-drop equations of Schmitz and Smith.[4]

Additional empirical data are required to firmly establish the form and Reynolds number limits of applicability of this correlation specifically for monopropellant hydrazine catalytic reactors. To further corroborate and broaden the range of applicability of this facet of the N_{PL} vs N_{Re} correlation while lacking such data, the author re-examined the older classical literature for flow through granular packed beds in general. Of particular interest were the more extensive data sets of Chilton and Coburn[5] covering a broad spectrum of granular packings and operating conditions. For the flow of a single compressible fluid through this type of bed, Leva[6] derived from first principles the equation

$$p_1^2 - p_2^2 = \frac{2\bar{z}RG^2\bar{T}}{g_c\bar{M}}\left[\ln\left(\frac{\rho_1}{\rho_2}\right) + \frac{2fL(1-\varepsilon)^{3-r}}{D_g\varepsilon^3\sigma^{3-r}}\right] \qquad (9)$$

where the subscripts 1 and 2 refer, respectively, to conditions upstream and downstream of the bed. Again, given a relatively small frictional pressure drop compared with the average absolute pressure, the density ratio in Eq. (9) is close to unity and the first term in the brackets is negligible relative to the second. Then with $r \approx 2$ and $p_1^2 - p_2^2$ written as $2\bar{p}\Delta p$, Eq. (9) can be rearranged to the same form as Eq. (8) and N'_{PL} and N_{Re_o} defined accordingly. Figure 3 shows the Chilton and Coburn data plotted in this format, spanning a three-decade Reynolds number range from about 2 to 2000. The data actually fall along a continuous curve but, for convenience, have been fitted with two regression lines corresponding to the viscous and turbulent regimes. The apparently smooth behavior in the transition regime ($N_{Re} \approx 10$ to 100) results from the averaging effect over myriad small flow passages where the onset of turbulence has occurred in some while viscous flow persists in others.

Vortex-Loss Metering Device

A convenient device for inserting a known and relatively high pressure drop in a liquid flow system, without the inherent susceptibility to contaminant plugging of a capillary tube or other small flow restriction, is a stacked-disk vortex-loss fluid resistor, such as the "Visco Jet."[7] Each disk or "stage" in the unit contains a multiplicity of tiny photoetched accelerate/decelerate spin chambers series-connected by passages of carefully controlled dimensions, as shown in Fig. 4. One such unit, as received from the manufacturer, was calibrated with the intended working fluid (hydrazine) over a range of temperature conditions. It was decided to employ the correlation technique discussed herein to facilitate conversion to operating conditions since the analytic form of the corrections required was not known. In defining the correlating parameters, the characteristic transverse linear dimension x was taken to be the minimum passage dimension x_p equivalent to its hydraulic diameter. The effective length L_e is not immediately apparent. The physical length of the unit could have been used, but a length related to the number of accelerate/decelerate chambers was considered more meaningful. Since the spin chambers in each stage are laid out essentially on a spiral, the flow path length through one stage should be proportional to the overall diameter of a stage D_S and for ν stages should be ν times as long.

Thus, taking the definitions

$$x_e = x_p \quad \text{and} \quad L_e = \nu D_S$$

the working form of the correlating parameters is

$$N_{Re} \equiv \frac{x_p G}{\mu}$$

and

$$N_{PL} \equiv \frac{g_c \rho x_p^3 \Delta p}{\nu \mu^2 D_S}$$

The calibration data are plotted in this form in Fig. 5 with negligible scatter. It is evident that over the observed range of Reynolds number ($N_{Re} \approx 250$ to 1300) these data are well represented by an equation of the form (5a)

$$N_{PL} = 0.25 N_{Re}^{2.37}$$

which is probably characteristic of the unique type of vortex flow loss induced in these devices. This relationship is extremely useful as either a pressure-drop predictor or as a flow-monitor calibration and is considerably more convenient than the treatments recommended elsewhere.

Conclusion

The success attained with the N_{PL}-N_{Re} correlation parameters in the three diverse applications cited tends to confirm the general utility of this approach for any uniformly distributed-loss device. Additional data in these and other applications are required to strengthen this tentative conclusion.

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